Calculating with fractions

The expectations are that at the end of year 3 pupils can;

- add and subtract fractions with the same denominator within one whole ,

$\frac{4}{5} - \frac{1}{5} =$ $\frac{1}{9} + \frac{4}{9} =$

The expectations are that at the end of year 4 pupils can;

- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same denominator
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths

4/5 + 3/5 =

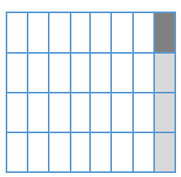
The expectations are that at the end of year 5 pupils can;

- add and subtract fractions with the same denominator and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

$\frac{3}{4} + \frac{7}{8} =$ $1\frac{1}{5} - \frac{1}{4} =$ $17 \times 1\frac{1}{2} =$

The expectations are that at the end of year 6 pupils can;

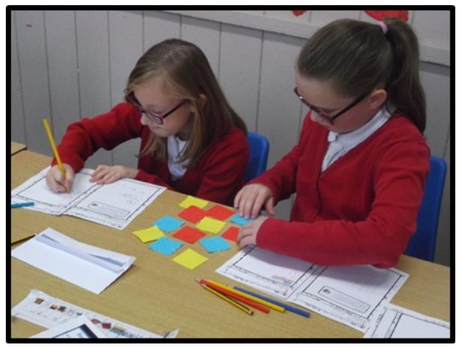
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form and showing understanding through diagrams
- divide proper fractions by whole numbers ,
- associate a fraction with division and calculate decimal fraction equivalents for example, 0.375] for a simple fraction

$\frac{1}{4} \times \frac{1}{8} =$  $\frac{2}{5} \div 2 =$ $\frac{3}{4} \div 2 =$

Park Junior School

Progress in calculation strategies

(In line with the New Curriculum 2014)



"Together we make a difference."

This policy has been written in response to the New National Curriculum September 2014, and aims to ensure consistency in the mathematical written methods and approaches to calculation across years 3-6.

The document is organised according to age related expectations, however it may be necessary for teachers to consult with other year groups in order to meet the needs of all children.

Wherever possible, it is important for teachers to create real life contexts for reasoning in maths.

As part of a child's learning in calculation, they need to be taught how to select the best method according to the numbers. The hierarchy of thinking should be;

DIVISION

Year 6

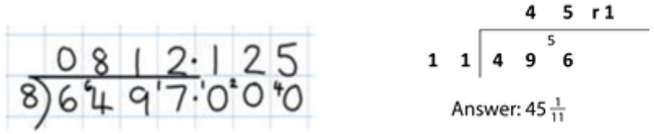
The expectations are that at the end of year 6 pupils can;

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve problems involving division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

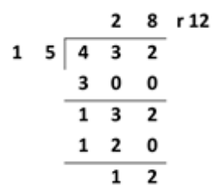
Key vocabulary

share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division,, 'carry', remainder, multiple, divisible by, factor, carry,

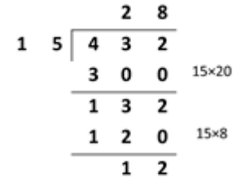
Divide numbers up to 4 digits by a two-digit number using the formal written method of short division



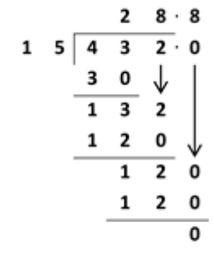
Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division



Interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context



$\frac{12}{15} = \frac{4}{5}$



Answer: 28·8

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DIVISION

The expectations are that at the end of year 5 pupils can;

- divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- divide whole numbers and those involving decimals by 10, 100 and 1000
- solve problems involving division including using their knowledge of factors and multiples, squares and cubes
- solve problems division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving division, including scaling by simple fractions and problems involving simple rates.

Key vocabulary

share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division,, 'carry', remainder, multiple, divisible by, factor, carry, remainder

Year 5

Become fluent in the formal written method of short division with remainders

Include carrying within the sum.

Remainders to be given

$$6\overline{)28^{4}} \quad 4\ 7\ r\ 2$$

as decimals and fractions depending on context

Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding

98 ÷ 4 = = 24 r 2 = 24 = 24.5 ≈ 25

Look at sums where the first number can't be divided into

$$4\overline{)35^{1}7} \quad 1\ 4\ r\ 1$$

Look at a number with 0s in

$$4\overline{)809} \quad 2\ 0\ 2\ r\ 1$$

Rationale for lower KS2

The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.

At this stage, pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value. Teaching should also ensure that pupils draw with increasing accuracy and develop mathematical reasoning so they can analyse shapes and their properties, and confidently describe the relationships between them. It should ensure that they can use measuring instruments with accuracy and make connections between measure and number.

By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.

Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.

Rationale for Upper KS2

The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.

At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems. Teaching in geometry and measures should consolidate and extend knowledge developed in number. Teaching should also ensure that pupils classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them.

By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

Pupils should read, spell and pronounce mathematical vocabulary correctly.

ADDITION

Year 3

The expectations are that at the end of year 3 pupils can;

- add mentally including three-digits and ones, three digits and tens and three digits and hundreds
- add numbers with up to three digits, using formal written methods of column addition
- estimate the answer to a calculation and use inverse to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition

Key vocabulary add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact

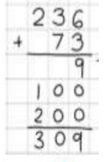
Use this intermediate step if children experience difficulty moving on from partitioning method straight to compact method

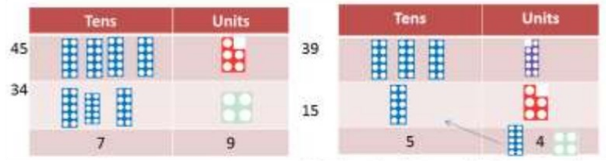
When do we know children are ready for this method?

Do they know addition and subtraction facts to 20?

Do they understand place value and can they partition numbers?

Can they explain their mental strategies orally and record them including informal jottings?





236

+ 73

309

1

Add the units first, carry numbers under-neath the bottom line, remind pupils of actual value eg, 3 tens add 7 tens.

Children who are very secure and confident with 3-digit expanded column addition, can be moved onto the compact column addition method, involving carrying. A comparison of the partitioning addition method to compact method is useful to show minimising the number of steps involved.

DIVISION

Year 4

The expectations are that at the end of year 4 pupils can;

- recall division facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- become fluent in the formal written method of short division with exact answers

Key vocabulary

share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division,, 'carry', remainder, multiple, divisible by, factor,

Become fluent in the formal written method of short division with exact answers.

98 ÷ 7 becomes

$$7\overline{)98} \rightarrow 14$$

Emphasise this is 20 not 2. Use dienes to show method

Answer: 14

Move on to carrying within the sum.

Look at sums where the first number can't be divided into

$$4\overline{)172} \qquad 4\overline{)358} \rightarrow 14$$

Look at 3 or 4 digits divided by 1 digit

Look at a number with 0s in

$$3\overline{)8^{2}5\,^{1}4\,^{2}1} \rightarrow 2847 \qquad 4\overline{)808} \rightarrow 202$$

DIVISION

The expectations are that at the end of year 3 pupils can;

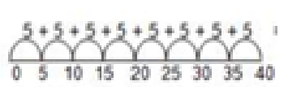
- recall and use division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving division including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Key vocabulary

share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, **inverse, short division, , 'carry', remainder, multiple**

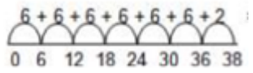
Year 3

Using a number line without a remainder



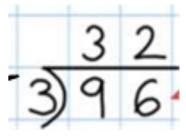
= 8 lots of 5

Using a number line with a remainder



= 6 lots of 5 and remainder of 5

Move to short division with no remainders or carrying



ADDITION

The expectations are that at the end of year 4 pupils can;

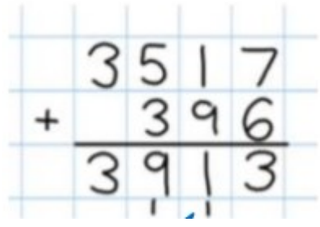
- add numbers with up to 4 digits using the formal written methods of column addition where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition two-step problems in contexts, deciding which operations and methods to use and why

Key vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, in- crease, vertical, 'carry', expanded, compact, thousands, hundreds, digits, inverse

Year 4

Add the units first



Carry numbers underneath

Remind pupils of actual value eg, 1 ten add 9 tens

Children use and apply this method to money and measures.

ADDITION

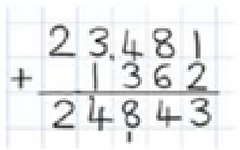
Year 5

The expectations are that at the end of year 5 pupils can;

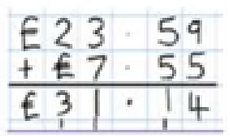
- add whole numbers with more than 4 digits, including using formal written methods (Column method)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determin, in the context of a problem, levels of accuracy
- solve addition multi-step problems in contexts, deciding which operations and methods to use and why

Key vocabulary

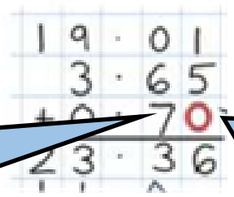
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, in- verse, decimal places, decimal point, tenths, hundredths, thousandths.



Use column addition to add two or three whole numbers.



Use column addition to add any pair of two place decimal numbers including amounts of money.



Say 6 tenths and 7 tenths to reinforce place value

Empty decimal places can be filled with a zero to show the place value of each column

Children should understand the place value of tenths and hundredths and use this to align numbers with differing numbers of decimal place.

MULTIPLICATION

Year 6

The expectations are that at the end of year 6 pupils can;

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve problems involving multiplication
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Key vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, tens, units, value, inverse, square, factor, integer, decimal, short/long multiplication, carry, tenths, hundredths, decimals

Use short multiplication to multiply decimals

```
  35.41
X     4
-------
141.64
 2 1
```

They undertake mental calculations with increasingly large numbers and more complex calculations.

Use long multiplication , multiplying 2 digits by up to 4 digits

```
  5341
X
  32
-----
 10682
16012 30
-----
170912
 1
```

Pupils explore the order of operations using brackets;

2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.

Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.

Carrying throughout the calculation should be introduced

Year 5 MULTIPLICATION

The expectations are that at the end of year 5 pupils can;

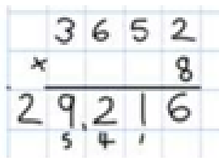
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non -prime) numbers. Establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply numbers mentally drawing upon known facts
- multiply whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}). Solve problems involving multiplication including using their knowledge of factors and multiples, squares and cubes
- solve problems involving multiplication , including understanding the meaning of the equals sign
- solve problems involving multiplication including scaling by simple fractions and problems involving simple rates.

Key vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, tens, units, value, inverse, square, factor, integer, decimal, short/ long multiplication, carry

Develop skills in short multiplication to 4 digits x 1 digit

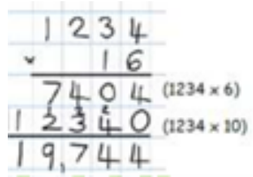
```
  3541
X    4
------
 14164
  2 1
```



Introduce long multiplication, multiplying 2 digits by 2 digits increasing up to 4 digits

```
  41
X 32
-----
  82
1230
-----
1312
 1
```

Carrying in addition stage only to start



Understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements

4 x 35 = 2 x 2 x 35;
3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10).

Use and explain the equals sign to indicate equivalence, including in missing number problems

13 + 24 = 12 + 25; 33 = 5 x ☐

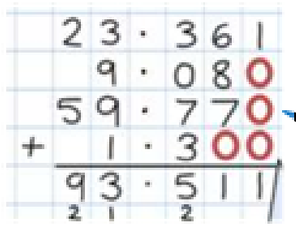
Year 6 ADDITION

The expectations are that at the end of year 6 pupils can;

- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition multi-step problems in contexts, deciding which operations and methods to use and why
- Add more complex numbers using the formal column method, including decimals

Key vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, in- verse, decimal places, decimal point, tenths, hundredths, thousandths.

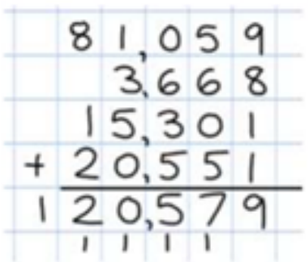


Tenths, hundredths and thousandths should be correctly aligned with the decimal point aligned vertically, including the answer

Empty decimal places can be filled with a zero to show the place value of each column

Use compact column method to add in context of money, measures, including decimals with different numbers of decimal places

Children should apply their knowledge of a range of mental strategies, mental recall skills, informal and formal written methods when selecting the **appropriate method** to work out addition problems. Opportunities to discuss the appropriateness of methods need to be planned for.



Year 3 SUBTRACTION

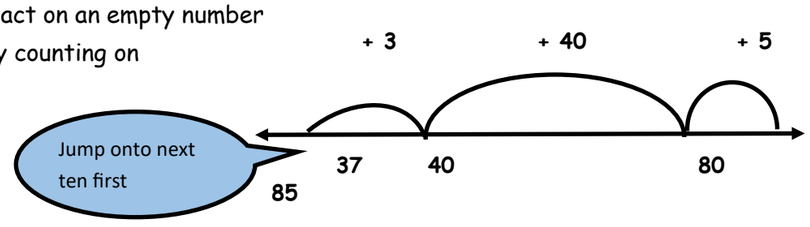
The expectations are that at the end of year 3 pupils can;

- subtract mentally including three-digits and ones, three digits and tens and three digits and hundreds
- subtract numbers with up to three digits, using formal written methods of column addition
- estimate the answer to a calculation and use inverse to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition

Key vocabulary

Equal to, take, take away, less, minus, subtract, leaves, tance between, how many more, how many fewer/less than,most, least,count, how many left, how much less is…, difference, count on, strategy, parti- tion, tens units, take and make, exchange, digit, value, hundreds

Subtract on an empty number line by counting on



Children should understand when to count back where appropriate, using place value or number facts. This skill should be reinforced through mental work.

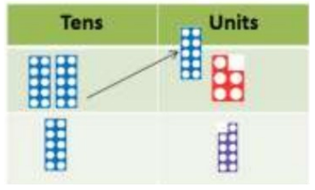
Begin to use formal column subtraction method, first using 'friendly numbers'

Friendly numbers, no exchange needed

$$\begin{array}{r} 38 \\ -\ 12 \\ \hline 26 \end{array}$$

Move to formal subtraction using .'take and make'.

Teach the children to consider the most appropriate method



$$\begin{array}{r} {}^{5}\not{6}\,{}^{1}3 \\ -\ 1\ 5 \\ \hline 4\ 8 \end{array}$$

Year 4 MULTIPLICATION

The expectations are that at the end of year 4 pupils can;

- recall multiplication facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Key vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, tens, units, value, inverse

Use the grid method to multiply 2 digit by 1 digit **if needed**

136 x 5 =680

X	100	30	6
5	500	150	30

500 + 150 + 30 = 680

Develop skills in short multiplication; 2 digit and 3 digit numbers x 1 digit numbers.

$$\begin{array}{r} 324 \\ \times\ \ 4 \\ \hline 1296 \\ \ \ 1 \end{array}$$

Use the distributive law;

39 × 7 = 30 × 7 + 9 × 7

and associative law

(2 × 3) × 4 = 2 × (3 × 4)

They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations

2 x 6 x 5 = 10 x 6 = 60.

Practise mental methods and extend this to three-digit numbers to derive facts,

600 ÷ 3 = 200 can be derived from 2 x 3 = 6).

Year 3 MULTIPLICATION

The expectations are that at the end of year 3 pupils can;

- recall and use multiplication for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Key vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, tens, units, value

Use the grid method to multiply 2 digit by 1 digit if needed

23 x 8 =184

X	20	3
8	160	24

160 + 24—184

Start with 2 digit x 1 digit without carrying then move on to simple carrying

Pupils develop efficient mental methods, for example, using commutativity and associativity

```
  3 2
X   3
-----
  9 6
```

4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240

Use multiplication and division facts to derive related facts

3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3

30 × 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3

```
  2 3
X   6
-----
1 4 8
  1
```


Year 4 SUBTRACTION

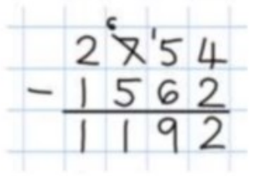
The expectations are that at the end of year 4 pupils can;

- subtract numbers with up to 4 digits using the formal written methods of column subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve subtraction two-step problems in contexts, deciding which operations and methods to use and why

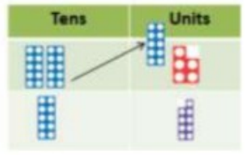
Key vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least, count-back, how many left, how much less is…, difference, count on, strategy, partition, tens units, take and make, exchange, digit, value, hundreds, inverse

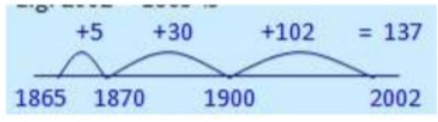
Subtract using formal column subtraction, using take and make where appropriate



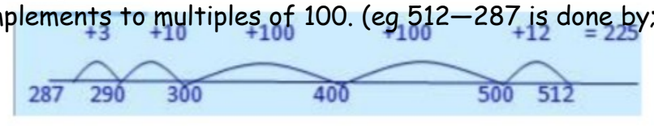
Use Numicon and Dienes to provide visual image for 'take and make'



Use complimentary addition to subtract amounts of money and for subtractions where the larger number is a near multiple of 1000 pr 100



Use counting up with confidence to solve most subtractions, including finding complements to multiples of 100. (eg 512—287 is done by;



Children should be encouraged to progress to using the fewest number of jumps.

SUBTRACTION

Year 5

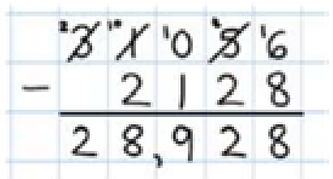
The expectations are that at the end of year 5 pupils can;

- Subtract whole numbers with more than 4 digits, including using formal written methods (Column method)
- Subtract numbers mentally with increasingly large numbers
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Key vocabulary

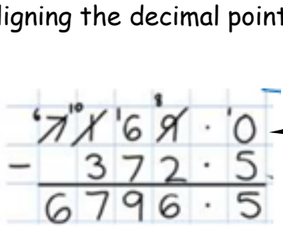
Equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least, countback, how many left, how much less is…, difference, count on, strategy, partition, tens units, take and make, exchange, digit, value, hundreds, inverse, tenths, hundredths, decimal point, decimal

Use compact column subtraction to subtract numbers with up to 5 digits



Use counting on for subtractions where the larger number is a multiple or near multiple or for decimals.

Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal point.



Put a zero in any empty decimal place to aid understanding of what to subtract

SUBTRACTION

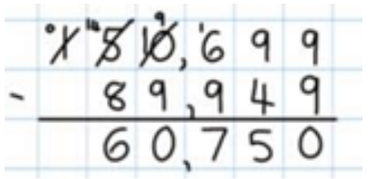
Year 6

The expectations are that at the end of year 6 pupils can;

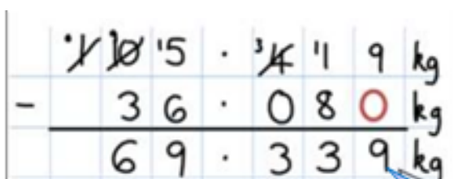
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- subtract more complex numbers using the formal column method, including decimals

Key vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least, countback, how many left, how much less is…, difference, count on, strategy, partition, tens units, take and make, exchange, digit, value, hundreds, inverse, tenths, hundredths, decimal point, decimal



Use compact column subtraction method to subtract more complex integers



Use compact column method to subtract in context of money, measures, including decimals with different numbers of decimal places

Children should apply their knowledge of a range of mental strategies, mental recall skills, informal and formal written methods when selecting the **appropriate method** to work out subtraction problems. Opportunities to discuss the appropriateness of methods need to be planned for.

Empty decimal places can be filled with a zero to show the place value of each column